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Energy of a system formed by a convective fluid and its container

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The Nusselt number and the energy content of a convective fluid and its container have been measured for two structures formed at low Rayleigh numbers. The results for the energy content are discussed. It is found that this energy is accumulated mainly in the lateral walls of the container, these being parallel to the rolls formed.

1. Introduction

Research into non-turbulent natural convection is mainly based on a series of experimental parameters (Koschmieder 1974) which are rather limited considering the complexity of the phenomenon. Consequently, it is desirable to study and measure new experimental parameters that can shed further light on the problem. This paper is an attempt to add a new experimental quantity to those classically used in convective systems, namely the energy content (Benofy & Quay 1983), which is a theoretical extension of the concept of internal energy to the steady states of non-equilibrium systems. In a convective fluid this quantity is measured with respect to the resting state, both being submitted to the same temperature difference. A description of the physical importance of the measurement of this quantity as well as some initial results can be found in Zamora & Rey de Luna (1984).

The generalized first principle of thermodynamics to steady states can be expressed as

$$U_{\mathrm{f}}-U_{\mathrm{i}}=-\int_{\mathrm{i}}^{\mathrm{f}}\left(\oint_{\Omega} J_q\,\mathrm{d}S\right)\mathrm{d}t-W_{\mathrm{ext}}, \tag{1}$$

where $U_{\mathrm{f}}-U_{\mathrm{i}}$ represents the difference of energy content between the initial (i) steady state and the final steady state (f), J_q the heat flux density vector, $\mathrm{d}S$ the elemental area, Ω the total surface area of the system, W_{ext} the work done on the system during the process and t the time.

It can easily be demonstrated that the mechanical energy accumulated by the fluid when it passes from a steady state of conduction to that of convection is too small to be detected experimentally. Thus, the energy-content increase measured between both steady states of the system is mainly due to an accumulation of a thermic nature. For the processes to be considered here the term W_{ext} can be neglected. The principles of the measurements and experimental techniques used are oriented towards the determination of the heat exchanges, i.e. the first term of the right-hand side of (1). A detailed description of the measuring principles and techniques is given in Zamora & Rey de Luna (1985). The present work describes the physical system studied, presents the measurements obtained, including both the Nusselt number and the energy content, and ends with an examination of the meaning of the new quantity introduced and an explanation of the nature of the thermal effects.

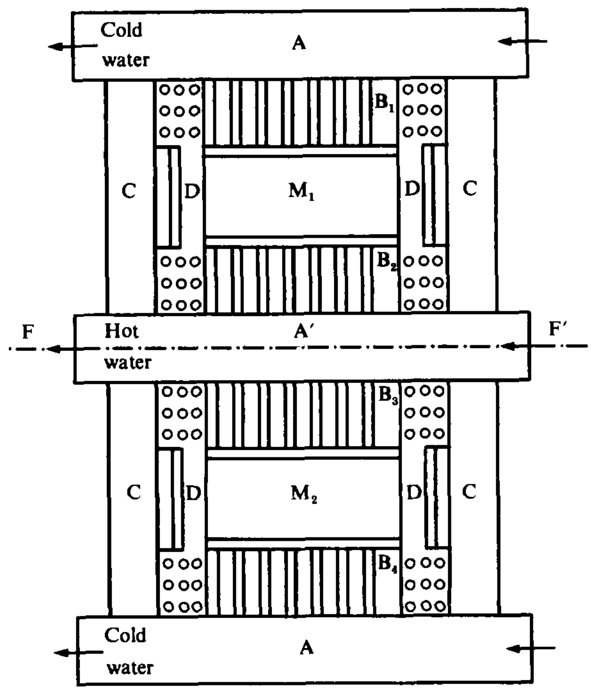

FIGURE 1. Sketch of the experimental apparatus. A and A′ are the pieces thermostated by circulating cold and hot water respectively. B_i are the heat fluxmeters M_1 the sample and M_2 the reference sample. D is the air chamber with baffles, C the guard ring and ○○○ the insulating material. FF′ is the axis of rotation. The lower part constitutes the control system.

2. Experimental

The sample studied was a parallelepiped of silicon oil Rhodorsil 47V350 (nominal viscosity 350 cS; viscosity at work temperature 240 cS; thermal conductivity $1.70\ 10^{-3}$ W/K cm; specific heat 1.66 J/g K and diffusivity $1.10\ 10^{-3}\ \mathrm{cm^2/s}$) with lateral dimensions $1.2H \times 2H$ and depth H of 1.96 cm. The silicon oil is contained laterally by means of Plexiglas walls (thermal conductivity $1.95\ 10^{-3}$ W/K cm; specific heat 1.46 J/g K and diffusivity $1.13\ 10^{-3}\ \mathrm{cm^2/s}$), 0.9 cm thick, in one corner of which a fluid dilator is placed. The thickness of the lateral walls of the box is chosen to reduce the lateral heat losses, to include the fluid dilator and to give the apparatus a mechanical rigidity. The floors of the parallelepiped are formed by two copper plates 0.2 cm thick, in which four thermocouples connected in series are placed. These measure the temperature difference applied to the fluid. The system, once filled and sealed adequately, forms a hermetically closed box of variable volume.

The sample formed by the fluid and its container is placed in the measuring system between two heat-flux meters (Moreno *et al.* 1980) each of which is kept in contact on its opposite face with a thermal source (see figure 1). A similar control system with a parallelepiped of solid Plexiglas instead of the sample was used to eliminate the thermal noise of the water of thermostation baths. In this way, the heat-flux meters which surround the sample measure the heat that, per unit time, enters and leaves the sample. By connecting the electrical output of the heat-flux meters in opposition, the difference of the heat fluxes which enter and leave can be measured directly, that is the accumulation or transfer of energy which, per unit time, the sample undergoes.

The measurement of the heat fluxes in the steady state were obtained with a standard error of 0.15 mW produced mainly by the thermal noise of the thermostats. The measurements of the energy content were obtained by connecting four heat-flux meters in pairs and in opposition to each other, two surrounding the sample, and two surrounding the control system. In this way a standard error of 0.04 mW for the accumulation power is obtained. Thus the estimated error for the energy-content measurements are 0.2 J, taking into consideration the duration of the process.

The experimental set-up has a horizontal axis which can be turned through 180° so that the sample can be inverted together with its thermal constraints. In this way it becomes possible to measure the heat fluxes interchanged between the sample and the thermal sources, as well as its instantaneous difference, both in the steady state (heated from below or from above) and during the transient state produced by the turn.

In the steady state, the differences between the heat fluxes which enter and leave the sample correspond to the lateral heat loss. An attempt was made to minimize these by adding a lateral guard ring to the system, separated from the central measuring unit by an air chamber of a lateral width of about 1 cm. A lateral vacuum chamber was not used because of the high vapour pressure of the fluid. However, in spite of this, owing to the change of the equivalent thermal conductivity of the fluid when the sample passes from the conduction to the convection state, it is always necessary to make corrections to eliminate the heat losses (Zamora & Rey de Luna 1985).

With our apparatus two kinds of measurements can be made. The first is of the heat fluxes in the steady state, together with the temperature differences. Thus, the Nusselt number can be determined and its dependence on the Rayleigh number analysed. The second studies the variation of the energy content, making use of the process produced by the inversion of the sample. For this type of measurement, it should be pointed out that when studying convective structures that are formed spontaneously from the quiescent state by an inversion of the system, it is possible to measure both the formation energy and the destruction energy. The formation energy of a convective structure is the variation of the energy content which accompanies the transformation of the system from the quiescent state to the convective state. Destruction energy corresponds to the reverse transformation of the system. This can be explained by the following diagram:

$$\begin{array}{c}\text{spontaneous}\\ \text{steady}\\ \text{convective}\\ \text{state}\end{array} \xrightarrow[\text{destruction}]{\text{turn } 180^\circ} \begin{array}{c}\text{quiescent}\\ \text{state}\end{array} \xrightarrow[\text{formation}]{\text{turn } -180^\circ} \begin{array}{c}\text{spontaneous}\\ \text{steady}\\ \text{convective}\\ \text{state}\end{array}$$

When determining the energy content of a structure whose formation is not spontaneous, but which requires some kind of induction or special manipulation to form it, only the destruction energy can be measured with sufficient precision, as shown below:

$$\begin{array}{c}\text{induced steady}\\ \text{convective state}\end{array} \xrightarrow[\text{destruction}]{\text{turn } 180^\circ} \begin{array}{c}\text{quiescent}\\ \text{state}\end{array}$$

In the present work both measurements have been performed, using a differential interferometric system for the observation of the convective structures (Bergé & Dubois 1979).

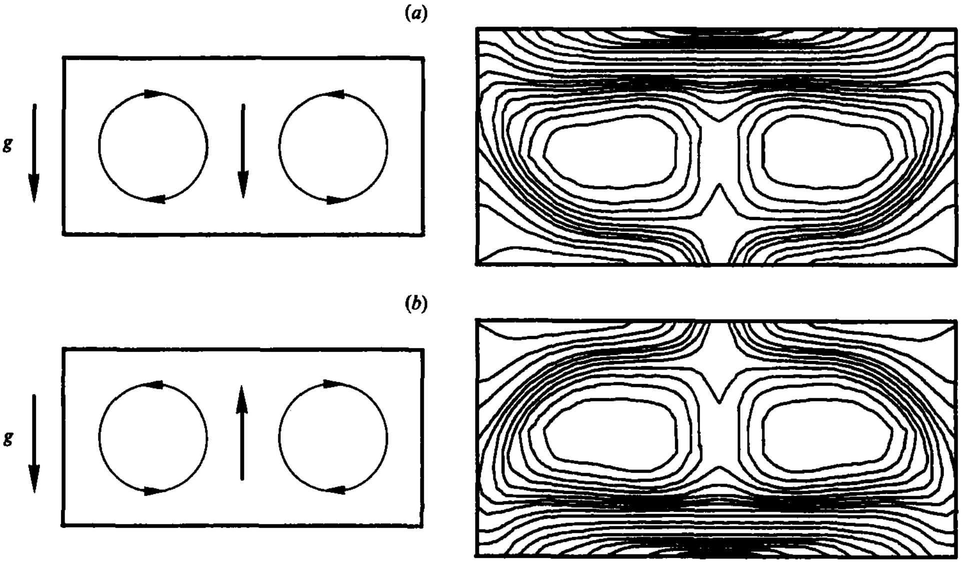

FIGURE 2. Diagrams of the convective structures formed and of their interferometric images: (a) induced structure; (b) spontaneous structure.

Spontaneous structure				Induced structure			
ΔT(K)	Ra/Ra_c	Nu	E(J)	ΔT(K)	Ra/Ra_c	Nu	E(J)
13.52	14.2	2.974	—	12.85	13.5	2.903	—
12.77	13.4	2.919	—	12.54	13.2	2.882	—
12.19	12.8	2.873	10.6	12.23	12.9	2.862	—
11.97	12.6	2.864	—	11.06	11.6	2.783	−10.3
11.42	12.0	2.816	10.0	10.13	10.7	2.714	−8.9
10.41	11.0	2.749	—	9.78	10.3	2.689	—
9.84	10.4	2.693	8.7	9.33	9.8	2.652	−7.7
9.08	9.6	2.631	8.0	8.53	9.0	2.588	−7.7
8.30	8.7	2.564	7.0	8.24	8.7	2.559	—
7.51	7.9	2.491	6.2	8.08	8.5	2.542	—
6.73	7.1	2.415	5.5	7.78	8.2	2.513	−6.5
5.10	5.4	2.22	4.0	6.98	7.3	2.442	−6.3
4.28	4.5	2.09	3.1	6.51	6.9	2.385	−5.5
4.07	4.3	2.06	—	6.16	6.5	2.348	—
2.67	2.8	1.75	1.5	6.06	6.4	2.338	−5.4
1.82	1.9	1.46	—	5.72	6.0	2.30	—
1.56	1.6	1.33		5.24	5.5	2.24	−4.8
1.35	1.4	1.21		4.85	5.1	2.18	—
1.19	1.3	1.1		4.17	4.4	2.06	−3.1
—	—	—		3.25	3.4	1.89	−2.3
—	—	—		2.93	3.1	1.79	—
—	—	—		2.58	2.7	1.72	—
—	—	—		2.24	2.4	1.61	—
—	—	—		1.93	2.0	1.49	—

TABLE 1. Mean values of Nusselt number Nu and energy content E versus temperature difference ΔT. The estimated Nusselt number error is $0.018/\Delta T$(K) and energy content error is 0.2 J. $Ra_c = 2500 \pm 170$.

3. Phenomenological aspects

During the initial process of testing the apparatus it was found that the air contained in the chamber that separates the central measuring unit and the lateral guard ring initiated natural convection at around $\Delta T = 5$ K. When this happened, the convective pattern in the fluid consisted of three rolls parallel to the short side of the box. These rolls are formed without a significant energy exchange, and persist for several hours until they degenerate into two rolls. At that moment, the process of absorption of energy by the sample was activated. The convection of air was avoided by introducing insulating solids into the chamber at the level of the flux meters and baffles of Plexiglas at the level of, and parallel to, the sample faces.

Once the lateral perturbations were eliminated, the fluid formed two well-defined horizontal rolls parallel to the short lateral dimension at low Rayleigh numbers. The first pattern had a central stream of fluid that was hot and ascending, while two downward flows were produced at the sides. This structure formed spontaneously from the quiescent state by an inversion of the sample. The second pattern was formed with a cold downward stream in the centre and two upward flows at the sides. The formation of this pattern did not arise spontaneously from the quiescent state by means of a simple inversion. To induce it, the equilibrium state was established and then the structure was produced by a brusque chilling of the upper floor of the sample. Figure 2 shows schematically both structures and the corresponding differential interferometric figures. In the latter, the deformation that the vertical isogradient temperature lines undergo along the rim of the box can be seen.

4. Results

First the results of the measurements of the Nusselt number

$$Nu = q_{\text{conv}}/q_{\text{cond}}$$

are presented. In our experiments the heat fluxes which go through the fluid sample in both conduction and convection steady states are measured independently.

The Nusselt numbers obtained for both the spontaneous structure and the induced one (table 1) were dealt with together. The fit to a formula proposed by Busse (1967),

$$\frac{(Nu-1)\,\Delta T}{\Delta T-\Delta T_{\text{c}}} = \left\{\frac{\Delta T-\Delta T_{\text{c}}}{\alpha}\right\}^{\beta},$$

confirms that this holds for all of our measurements, with the following values:

$$\begin{aligned} \beta &= 0.31 \pm 0.01; \\ \alpha &= (1.21 \pm 0.06)\ \text{K}; \\ \Delta T_{\text{c}} &= (0.95 \pm 0.01)\ \text{K}. \end{aligned}$$

ΔT_{c} corresponds to $Ra_{\text{c}} = 2500 \pm 170$ (figure 3). The exponent β is found to be within the limits given by numerical computations (Busse 1978), while the value found for Rayleigh's critical number is in agreement with the results obtained from numerical calculations based on models (Frick & Clever 1982).

The second quantity studied is the energy content of the convective structures in the quiescent state. In general, during a process carried out by the system connecting two steady states, the instantaneous sample absorption power

$$-\oint_{\Omega} J_q \,\mathrm{d}S$$

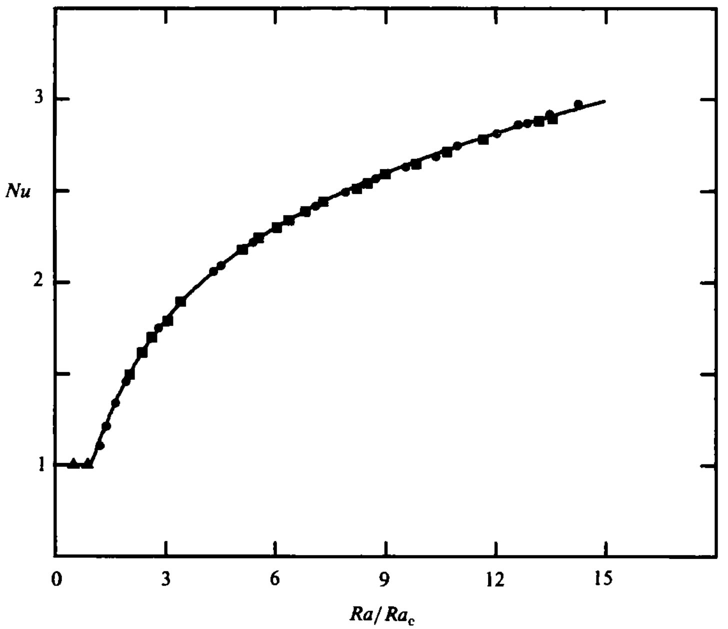

FIGURE 3. Nusselt number versus relative Rayleigh number: ▲, quiescent state; ●, spontaneous structure; ■, induced structure.

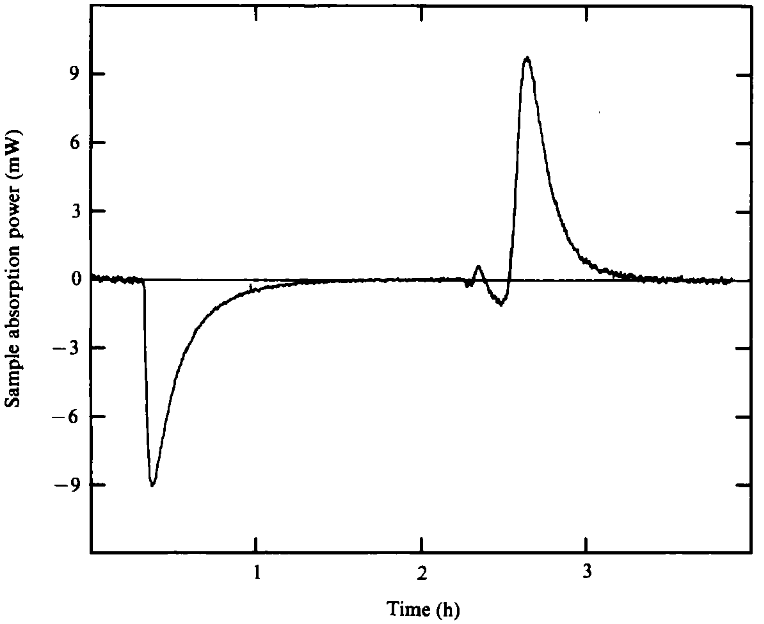

FIGURE 4. A typical thermograph of the processes of destruction and formation of the spontaneous structure.

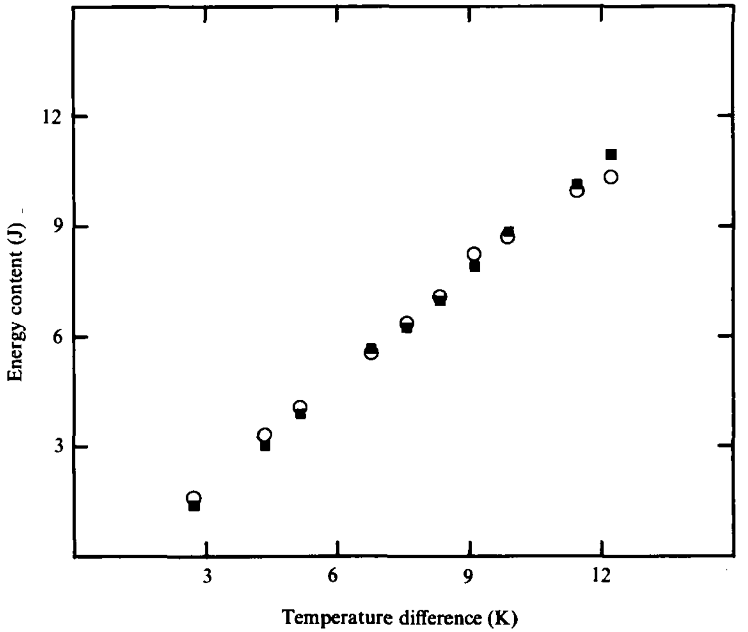

FIGURE 5. Energy content of the spontaneous structure versus the temperature difference applied: ■, value measured during destruction; ○, value measured during formation.

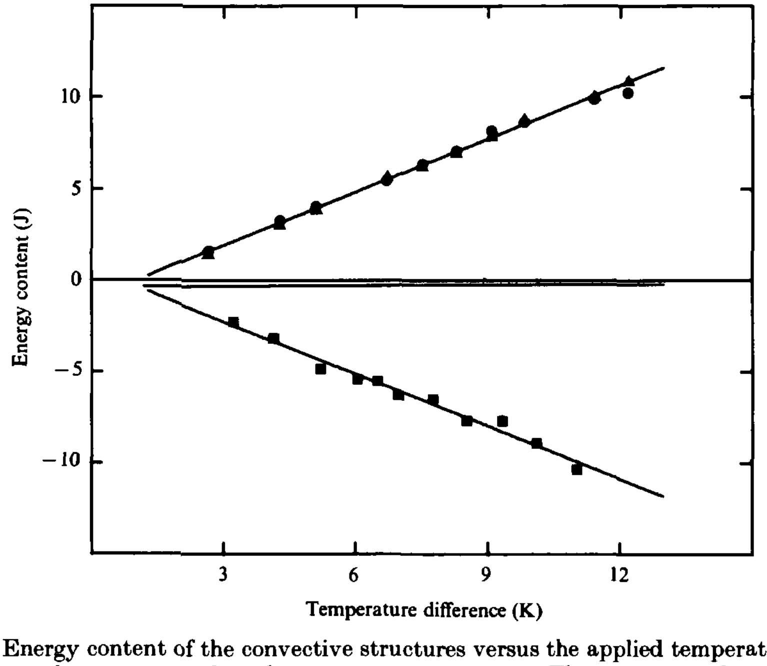

FIGURE 6. Energy content of the convective structures versus the applied temperature difference. The positive values correspond to the spontaneous structure. The negative values correspond to the induced structure. The central line represents the algebraic sum of the two adjustments.

is measured by an appropriate electrical connection between the heat-flux meter output (Zamora & Rey de Luna 1985). A time integration furnishes the energy-content difference between both steady states. The measurements were made in two steps. The first studied the energy in both the destruction and the formation processes of the spontaneous structure. The second studied the energy in the destruction process of the induced structure.

Figure 4 shows a typical corrected thermograph of the processes of destruction and formation carried out during the first stage. Integration of the thermograph permits the determination of the energy content associated with the two processes under study, the results being shown in table 1 and represented in figure 5. Within the limits of experimental error both energy contents are equal and, therefore, functions of the steady state (Benofy & Quay 1983), following the formula

$$E = -1.00 + 0.970\,\Delta T,$$

where ΔT is expressed in °C and E in Joules.

Once it had been demonstrated that the energy content is a steady state function, we proceeded to determine the corresponding energy content of the induced structure, using the destruction process alone. The results are shown in table 1 and are represented in figure 6. The experimental points follow the formula

$$E = 0.66 - 0.958\,\Delta T.$$

Their spread can be attributed to the thermal fluctuations of the thermostatic baths used in this second set of experiments. As before, ΔT is expressed in °C and E in Joules.

As can be seen, the induced convective structures give an absolute value for the energy content which very closely resembles that of the spontaneous structure, the differences falling within the margin of experimental error.

5. Discussion

This discussion deals with the analysis and physical interpretation of the results obtained for the energy content and with the study of transitory situations.

The energy, a steady-state function, should become apparent by means of an average temperature change of the sample, as this lacks the capacity for mechanical accumulation. From a thermal point of view, it is surprising that a system which is so highly constrained produces this twofold behaviour of the energy (figure 6). It was expected that, instead of this antisymmetrical energetic behaviour, a symmetrical one would be found, in agreement with the Bénard–Rayleigh problem, when the infinite lateral dimension is studied for the Boussinesq case. In this situation, it was found necessary to do additional experiments which permitted the distribution of energy within the sample to be clarified.

The first experiment measured the average temperature variation of the fluid, which accompanies the change from the convective to the conduction state, for the same applied temperature difference. A thin tube (1 mm internal diameter) filled with the same fluid was connected to the box, using the tube as an expansion system of the fluid of the box, forming a dilatometer. Once the steady state of convection had been reached, the system was inverted to give the conduction state. In this way, the variation in the level of the liquid in the tube was a measurement of the variation of the average temperature of the fluid and, therefore, of the energy content exchanged. The variation in volume found, always less than 1 mm^3, alone accounted

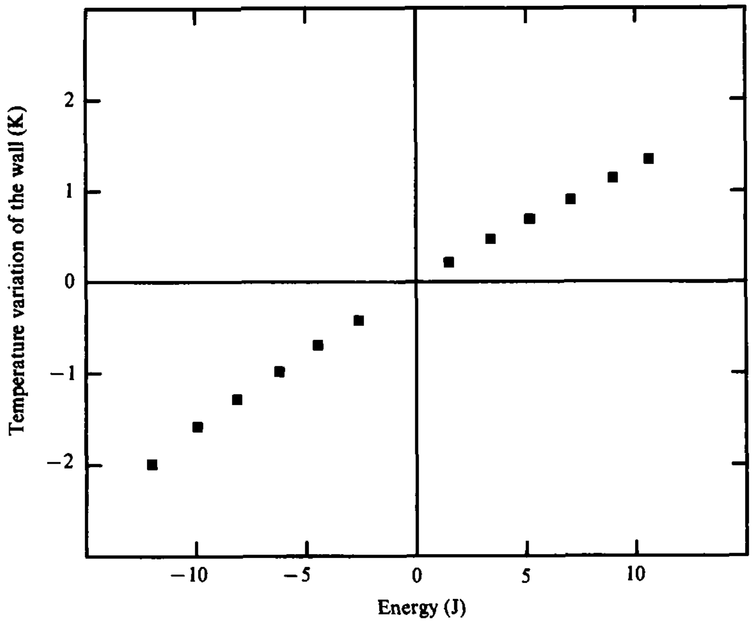

FIGURE 7. Variation of the temperature of the lateral wall during the change from the conduction to the convection states versus the energy content, for both structures.

for about 10 % of the energy content measured. This result is basically in agreement with the classical behaviour of the Bénard–Rayleigh problem.

Once the fluid had been eliminated as the main energy-accumulating agent, only the consequences of the lateral container remained to be analysed. Given that the convective structures formed by the fluid are two-dimensional and perpendicular to the longer lateral dimensions, it is expected that the average temperature of these walls stays the same as that of the fluid. Therefore the energy accumulation must mainly occur in the shorter lateral walls. This also accounts for the deformations which the interferometric lines show along the edges, see figure 2, which are clearly antisymmetrical. Another confirmation that the energy accumulates in the shorter lateral walls can be obtained from the heat losses of the sample, which are affected by the average temperature change of the walls. Given that our experimental system permits the resolution of these losses, their values, although very small (less than 1 % of heat flux), confirm qualitatively the hypothesis put forward. The lateral heat losses depend linearly on the applied temperature difference, being positive for the spontaneous structure and negative for the induced structure.

The second experiment measured the temperature change of the shorter wall when the system goes from a convection to a conduction state. A thermocouple was inserted in the geometrical centre of one of the faces about 0.5 mm from the internal surface. The measured temperature increases plotted against the energy content are shown in figure 7 for both convective structures. The temperature increases found show that the energy transfer occurs mainly in the walls. The nonlinear behaviour of the measured temperature increment may be accounted for by the measured temperature increment represented not being the average temperature increment of the wall. It is obtained from a point in the wall near the fluid where a local perturbation in the horizontal distribution of the wall temperature reaches the highest value. For this reason, the heat capacity equivalent to the lateral walls ($\mathrm{d}E/\mathrm{d}\delta T$) is approximately half of its capacity (14.3 J/K).

Therefore it is concluded that the energy content of the dissipative structures originates from the interaction between the fluid and its container, and that it accumulates preferentially on the lateral walls which are parallel to the rolls.

The transient situations can be studied directly from a thermograph, since this basically represents the energy exchange by the sample during a unit time. As can be observed, the destruction of the convective structure is triggered in a uniform fashion immediately after the inversion. On the other hand, the energetic process of formation occurred about fifteen minutes later when the final boundary layers are clearly formed. During the initial time period three imperfect rolls formed temporarily, which degenerated into two permanent rolls. The time characteristics of relaxation towards the final steady state are the same as the heat transmission through the Plexiglas walls.

The process of energy exchange occurs in transient situations. The form of energy exchange with the wall, absorption or transfer, and the type of convective structure formed, spontaneous or induced, are closely related. Actually, in the case of the spontaneous structure there is a descending fluid motion next to the wall which absorbs energy, and therefore, favours the downward motion. Inversely, the induced structure involves an ascending motion in the proximity of the wall, which transfers energy to the fluid and so assists the upward motion. Altogether, the energy exchange between the lateral wall parallel to the roll and the fluid in the initial transient situation is of such a nature that it tends to assist the motion.

From this it follows that, in the transition period of the convective structures, the lateral walls parallel to the motion act as additional thermal sources which exchange heat with the fluid. These transient heat interchanges occur in such a way as to destabilize the resting state, being opposite to that occurring with the mechanical interaction. This would account for the well-known fact that thermal convection is initiated in the proximities of the walls, and particularly in such a way that the motion is parallel to them (Koschmieder 1967).

Therefore it can be concluded that the condition of lateral adiabaticity imposed on the box is not met in the fluid in transient situations. If, on the other hand, the system does not fully satisfy the condition of external thermal isolation, the structure formed in the initial transient situation may remain fixed in the steady state. This may account for the structure being made up of three rolls which are always formed initially in our experimental set-up and which remain for hours when the external lateral air is kept in convection.

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